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Complementarity in Models of Public Finance and Endogenous Growth*

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Abstract

This paper considers the effects of complementarity in private production between private and public inputs on optimal fiscal policy under the objective of growth maximization. Using an endogenous growth model with public finance and CES technology, it derives two central results. First, it shows that with complementarity, growth-maximizing fiscal policy is also affected by preference parameters, the degree of complementarity and the stock-flow properties of public inputs to private production. Second, it shows that optimal public spending composition and taxation are interrelated and also depend on the efficiency of public spending under growth maximization. Both results contrast with standard findings in the literature that are typically based on the assumption of Cobb-Douglas technology, and have important lessons for policy settings.

JEL code: E62, H21, H50, O40

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1 Introduction

The objective of this paper is to examine the effects of complementarity between public and private inputs to private production on growth-maximizing fiscal policy, within the context of an endogenous growth model.[1] The role of input complementarity for economic development is increasingly appreciated. Early work by Kremer (1993), for example, argues that when private production requires a range of complementary inputs, the failure of one can have disastrous consequences (the so called 'O-ring'-theory). More recently, Temple (2009) has suggested that growth determinants are complements rather than substitutes, which can give rise to bottlenecks with a disproportionate negative impact on growth. Similarly, Jones (2011) shows that large differences in TFP across countries can only plausibly be explained by complementarity of inputs to private production among other factors. By contrast, the analysis of the effects of complementarity for fiscal policy in the context of growth models has received limited attention so far.

We develop a standard endogenous growth model with public finance where the government is assumed to levy a flat tax on income which distorts private investment. Similar to Tsoukis and Miller (2003), Ghosh and Roy (2004), and Agénor (2011) for example, we assume that the government uses the proceeds from this tax to finance productive public services and public capital which both enter private production thereby enhancing the productivity of the private sector. In addition, we also consider the efficiency of public spending, as in Agénor (2010), and we model the production of public services in greater detail similar to Agénor (2008b).

We then amend this framework by assuming that private and public inputs are complements through imposing a CES production function, contrary to most existing papers which are based on a Cobb-Douglas framework. As-

[1] Endogenous growth models with a fiscal policy dimension, such as Barro (1990) and Futagami et al. (1993), who derive growth-maximizing levels of taxation and public spending composition, have increasingly formed the basis for empirical investigation that fiscal policy affects long-run growth. See, for example, Acosta Ormaechea and Yoo (2012); Gemmell et al. (2012). Also, Arnold et al. (2011) examine tax composition effects on long-run GDP levels based on a augmented Solow model. This evidence confirms the predictions of endogenous growth models with public finance.

suming complementarity is in line with the notion that private inputs are at best very imperfect substitutes for public inputs in the production of private output. Devarajan et al. (1996) and Ghosh and Gregoriou (2008) are among the few papers that present an endogenous growth model with public finance based on CES technology. However, their models are not as rich as that presented below and include only two fiscal policy parameters. These previous CES-based models also ignore public capital and do not identify the *growth-maximizing* fiscal policy.

In this paper, we focus on the implications of complementarity for fiscal policy under growth maximization. For our purposes, considering growth maximization is interesting because there is a standard, and relatively uncontroversial, finding in the existing literature when the simple, but potentially unrealistic, case of Cobb-Douglas (C-D) technology is assumed. In the C-D case, endogenous growth models with public finance imply that only the factor share parameters of the production function for final output and, where included, of public inputs determine the growth-maximizing tax rate, the volume of public spending, and the growth-maximizing expenditure composition. As a result, the growth-maximizing levels of both the tax and public spending policy parameters are independent of each other.

By allowing for CES production technology, we generate a number of interesting results with respect to the optimal level and the optimal composition of public spending (under *the objective of growth maximization*) which contrast with the equivalent case under C-D technology.[2]

First, we show numerically that CES technology implies that the number of parameters that determine optimal fiscal policy under growth maximization significantly increases even though we leave all remaining features of the model unchanged. With CES technology the growth-maximizing fiscal policy is also determined by preference parameters, other technology parameters (in addition to the share parameters), and the stock-flow properties of public inputs (which can be interpreted as the rate of depreciation of public capital).

[2]There are obviously differences between growth- and welfare-maximizing fiscal policy. However, Misch et al. (2013) show that differences in outcomes in terms of growth and welfare under growth and welfare maximization are nevertheless fairly small despite the differences in policies.

These results suggest that the assumption of C-D technology understates the complexity of the various factors determining growth-maximizing fiscal policy in otherwise identical models.[3] [4]

Second, again using numerical methods, we show that there are instances where the growth-maximizing level of productive spending (which equals the level of taxation) depends on its composition and vice versa. In particular, we show that the optimal level of taxation is higher when the composition of public spending is suboptimal; and that the optimal share of public resources allocated to public investment may be very low when the level of taxation deviates from its first-best level. Similar results arise when public spending is not efficient. These are therefore additional, but simple and intuitive, cases of second-best interaction in public finance that have largely been ignored in the literature and that do not arise under C-D technology.

In practice, governments are likely to be constrained in their ability to alter either total public spending or its mix, implying that such model predictions may be relevant to the achievement of growth-maximizing fiscal policy in practice as well. On the expenditure side, such constraints may arise as a consequence of budget rigidities due, for example, to quasi-fixed expenditure items such as social welfare benefits linked to entitlement conditions, or interest payments that depend on the previously accumulated stock of public debt.[5] On the revenue side, such constraints may arise where there are large informal sectors that keep tax revenue levels relatively low, as in some developing countries. Frequently, these types of constraints can persist over relatively long periods of time.

The advantage of assuming C-D technology in endogenous growth models with public finance is, of course, that closed-form solutions for growth-

[3] The result is also an obvious analogy to the theory of taxation which demonstrates that even in simple static tax models, the optimal tax system depends on a wide range of factors for which it may be difficult to find empirical counterparts even when a range of simplifying assumptions is made (Creedy, 2009).

[4] Under welfare maximization, similar results may arise in the absence of CES technology.

[5] Budget rigidities may significantly constrain governments in practice: Mattina and Gunnarsson (2007) for instance estimate that the share of spending that is non-flexible due to legal obligations (which includes social benefits, interest payments, compensation of public employees, and subsidies) amounts to 72% in Slovenia.

maximizing fiscal policy exist. By contrast, closed-form solutions often cannot be obtained when CES technology is assumed. For this reason, we are forced to rely on numerical examples to derive our results. While the well-known drawback of numerical examples is that they cannot establish the generality of some results, they nevertheless serve the purpose of our paper. In particular, they suffice to show that under sensible parameter assumptions, the results with respect to growth-maximizing fiscal policy under C-D technology cannot be generalized and may cease to hold at least in some plausible cases.

Our model results have important implications for optimal fiscal policy setting, and our framework suggests that commonly held beliefs about what constitutes optimal fiscal policy under growth maximization may not be valid; at least where complementarity in production technology is important. For instance, our results suggest that with low levels of revenue collection (as in many developing countries), optimal public investment under growth maximization is low relative to a situation where this is not the case - even if the output elasticity of public capital differs significantly from zero. This result contrasts with common beliefs about the importance of public investment for long-run growth, but they are in principle consistent with Ghosh and Gregoriou (2008) who find that reallocating public resources in favor of public investment is growth-enhancing based on data from developing countries.

The remainder of this paper is organized as follows. Section 2 develops the model and derives the equilibrium of the market economy. Section 3 derives growth-maximizing fiscal policy with Cobb-Douglas technology as a benchmark case. Sections 4 and 5 demonstrate the effects of complementarity on optimal fiscal policy under growth maximization. Section 6 concludes.

2 The Model

The public finance growth framework we adopt in the paper is based on Devarajan et al. (1996). We extend their model by *simultaneously* considering public services and public capital as in Tsoukis and Miller (2003), Ghosh and Roy (2004), and in Agénor (2011) for example. We model the efficiency of public spending as in Agénor (2010), and a production function for public services in a similar way as Agénor (2008b) for example, together with CES technology as in the original model by Devarajan et al. (1996). We assume that there is a large number of infinitely lived households and firms that is normalized to one so that firm entry and exit cancel out, or are absent, and population growth is zero.

The representative firm produces a single composite good using private capital, k, broadly defined to encompass physical and human capital, and two public inputs, G_1 and G_2, based on CES technology:

$$y = (\theta k^{\upsilon} + \alpha_1 G_1^{\upsilon} + \alpha_2 G_2^{\upsilon})^{\frac{1}{\upsilon}} \tag{1}$$

where θ, α_1 and α_2 are share parameters with $\theta = 1 - \alpha_1 - \alpha_2$. The productivity of private capital used by the individual firm therefore depends positively on G_1 and G_2, which can be thought of as being provided by different government sectors (e.g. education and transport infrastructure). Private vehicles, for example, may be used more productively when the quality of the road network increases. G_1 and G_2 are non-rival and provided free of charge to the agents of the economy. The parameter υ determines the elasticity of substitution which corresponds to $\frac{1}{1-\upsilon}$. With $\upsilon = 0$, the production technology is Cobb-Douglas.[6]

G_1 denotes the amount of productive public services provided by the government (e.g. public law enforcement, public education services), whereas G_2 denotes the stock of public capital (e.g. public infrastructure) which the government accumulates through public investment, $\dot{G}_2$. In other words, G_1 can be interpreted as a public input to private production which fully depreciates over one period, and G_2 can be interpreted as a public input with infinite lifetime that does not depreciate at all. To capture the notion that factors of production are complements rather than substitutes, it is assumed that $\upsilon \leq 0$. This assumption seems justified when considering public inputs provided by the government which differ fundamentally from private inputs,

[6] We recognize that a more general specification of (1) would be a nested CES function that allows for different elasticities of substitution between G_1 and G_2 on the one hand and between G_1 and G_2 taken together and private capital on the other. However, for the purpose of this paper, our specification of the production function is sufficient.

such that it may be very costly for firms to substitute private alternatives for them. For instance, privately generating electricity is typically much more expensive than using electricity from the public grid.

The government finances total public expenditure by levying a flat tax, τ, on income, and the government budget is assumed to be always balanced. We further assume that the technical efficiency of public spending may vary. For instance, inefficiencies arise if the government purchases the inputs for G_1 and G_2 at a high price, or if there is waste due to corrupt bureaucrats. While changing the level of technical efficiency may also involve a resource cost, we refrain from modeling this in greater detail for simplicity because this is not needed to derive our main results in later sections.

G_1 itself is produced using two different inputs, G_A and G_B, which can be interpreted as sub-sectoral public spending categories, based on CES technology:

$$G_1 = (\omega G_A^{\varepsilon} + \beta G_B^{\varepsilon})^{\frac{1}{\varepsilon}} \tag{2}$$

with $\omega = 1 - \beta$ and where ε determines the elasticity of substitution. This feature of the model allows for a richer specification of fiscal policy because the *inter-sectoral* allocation of public resources (between G_1 and G_2), and the sub-sectoral allocation of public resources (between G_A and G_B) can be distinguished. It allows us to analyze the effects of misallocation at the sub-sectoral level on the growth-maximizing tax rate and the inter-sectoral composition below. Analogously to the production of final output, we assume that $\varepsilon \leq 0$, reflecting the notion that G_A and G_B, are complements. For simplicity, we set $\varepsilon = \upsilon$ which facilitates the derivation of the results but does not change them qualitatively. G_A may represent, for example, the amount of goods and services, and G_B represent spending on public administration, within the production of G_1.[7]

Let ϕ_1 (ϕ_2) determine the inter-sectoral allocation of public resources

[7] In addition, it would also be possible to model the production of public capital in greater detail. For simplicity, we refrain from this as our purpose is to show that the sub-sectoral allocation of public resources influences the growth-maximizing values of the tax rate and of the inter-sectoral allocation which we show further below. Obviously, the allocation of resources among different inputs to public capital is also likely to matter, but we do not pursue this here.

and denote the share of total public expenditure that is allocated to G_1 ($\dot{G}_2$) with $\phi_1 + \phi_2 = 1$ (i.e., the share of resources allocated to public investment is $\phi_2 = 1 - \phi_1$) and let ϕ_A (ϕ_B) denote the share of public spending on G_1 that is allocated to G_A (G_B) with $\phi_A + \phi_B = 1$. Further, let κ_1 and κ_2 denote the technical efficiency of public spending on G_1 and G_2 which we assume to be different from the allocative efficiency. G_j (with $j = A, B$) can therefore be written as

$$G_j = \kappa_1 \phi_1 \phi_j \tau y \tag{3}$$

Using (2) and (3), the amount of G_1 can therefore be written as

$$G_1 = \kappa_1 \phi_1 (\omega \phi_A^{\varepsilon} + \beta \phi_B^{\varepsilon})^{\frac{1}{\varepsilon}} \tau y \tag{4}$$

The level of public investment, $\dot{G}_2$, can be written as

$$\dot{G}_2 = \kappa_2 \phi_2 \tau y \tag{5}$$

We normalize k_i so that at $k_i = 1$ (with $i = 1, 2$), public spending is assumed to be perfectly efficient in a technical sense. For simplicity, we assume that increasing the efficiency of public spending is possible at no cost (i.e. increasing k_i does not involve a trade-off). While in principle, this means that governments would never choose any value for k_i below one in the absence of constraints, this assumption merely serves as a simplification and allows us to address the hypothetical question of what *would* happen if public spending was *not* perfectly efficient. However, to capture the notion that efficiency gains are limited, we assume that $\kappa_1 \leq 1$ and that $\kappa_2 \leq 1$.

The households own the firms and therefore receive all their output net of taxation which they either reinvest in the firms to increase their capital stock or use for consumption, depending on their preferences and the returns to private capital. Private investment by the representative household equals

$$\dot{k} = (1 - \tau) y - c \tag{6}$$

The representative household chooses the consumption path to maximize lifetime utility U given by

$$U = \int_0^{\infty} \left(\frac{c^{1-\sigma}}{1-\sigma} \right) e^{-\rho t} dt \tag{7}$$

subject to the household's resource constraint given by (6) taking τ, G_1, G_2 and $k_0 > 0$ as given.[8] From the first-order conditions, the growth rate of the household's consumption, and of the economy, can be written in familiar form as

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma}\left((1-\tau)y_k - \rho\right) \tag{8}$$

In order to ensure that the transversality condition holds and does not constrain the choice of τ and $\phi_{1,2}$, it is assumed that $\sigma > 1$.[9]

Along the balanced growth path, output can be expressed as

$$y = \frac{\dot{y}}{\gamma} \tag{9}$$

Using (9) to substitute for y in (5), and integrating, yields

$$G_2 = \frac{\kappa_2 \phi_2 \tau}{\gamma} y \tag{10}$$

For the remainder of this section, we assume Cobb-Douglas technology to simplify the analytical expressions. Hence, when $\upsilon = 0$ (and $\varepsilon = 0$), the production function can then be written as:

$$y = k^{\theta} G_1^{\alpha_1} G_2^{\alpha_2} \tag{11}$$

where $\theta = 1 - \alpha_1 - \alpha_2$. The marginal product of capital, y_k, can be written as:

$$y_k = \theta \left(\frac{G_1}{y}\right)^{\alpha_1} \left(\frac{G_2}{y}\right)^{\alpha 2} \left(\frac{y}{k}\right)^{\alpha_1 + \alpha_2} \tag{12}$$

Using (4), (10) and (11) to substitute for G_1/y, G_2/y and y/k in (12), and using (12) to substitute for y_k in (8) yields:

$$\gamma = \frac{1}{\sigma}\left((1-\tau)\theta \left((\phi_{1A})^{\omega} (\phi_{1B})^{\beta} \kappa_1 \phi_1 \tau\right)^{\frac{\alpha_1}{\theta}} \left((\kappa_2 \phi_2 \tau) \frac{1}{\gamma}\right)^{\frac{\alpha_2}{\theta}} - \rho\right) \tag{13}$$

[8] The time subscript is omitted whenever possible. A dot over the variable denotes its derivative with respect to time. The initial stock of public capital must also be greater than zero.

[9] The transversality condition can be written as $\lim_{t \to \infty} [\lambda k] = 0$ where λ is the costate variable of the current-value Hamiltonian.

Note that (13) is not a 'final' expression for the growth rate but merely an equation that the growth rate must satisfy because γ also appears on the RHS.

The Appendix shows that the equilibrium of the model is saddlepoint stable within relevant parameter ranges, and that the balanced growth path is unique. Along the balanced growth path, c, k, G_1, G_2 and y all grow at the same rate.

3 Optimal Fiscal Policy with Cobb-Douglas Technology under Growth Maximization

This section derives optimal fiscal policy when output (y) and public services (G_1) are produced using Cobb-Douglas technology under the objective of growth maximization. This benchmark case will allow us to demonstrate the role of complementarity in later sections.

For simplicity, we assume throughout the paper that the objective of the government is to maximize growth in contrast to papers that derive the welfare-maximizing fiscal policy in similar frameworks as Ghosh and Roy (2004) for example. While in these models, growth and welfare maximization are not identical, in practice, growth maximization is less complex and more common as changes in output are easier to observe than welfare. In addition, the differences in outcomes between growth and welfare maximization in similar models often appear to be small (see Misch et al., 2013). Below we use the term 'optimal' fiscal policy to refer to the growth-maximizing values of the tax rate and of the public spending shares of public services and public investment (denoted by τ^* and $\phi^*_{1,2}$, respectively).

Cobb-Douglas technology implies $\upsilon = \varepsilon = 0$. Since the model is based on the assumption that there is no cost to increased efficiency, the government sets $k_{1,2}$ at their maximum values $\kappa^*_{1,2}$ to ensure that public spending is fully efficient:

$$\kappa^*_{1,2} = 1 \tag{14}$$

(which obviously maximizes growth and welfare, and which does not depend on the underlying production technology). Implicitly differentiating (13) yields the growth-maximizing income tax rate, τ^*, which corresponds to:

$$\tau^* = \alpha_1 + \alpha_2 \tag{15}$$

and the growth-maximizing inter-sectoral expenditure shares, $\phi^*_{1,2}$, which correspond to

$$\phi^*_{1,2} = \frac{\alpha_{1,2}}{\alpha_1 + \alpha_2} \tag{16}$$

where $\phi^*_1 + \phi^*_2 = 1$. Further, the growth-maximizing sub-sectoral expenditure shares within G_1, $\phi^*_{A,B}$, correspond to

$$\phi^*_A = \omega \tag{17}$$

and

$$\phi^*_B = \beta \tag{18}$$

Hence (15), (16), (17) and (18) suggest that with Cobb-Douglas technology, the growth-maximizing tax rate and expenditure shares depend only on share parameters of the production functions of final output and of public services. These results correspond to those derived in the existing literature: for example, Agénor (2008a), Agénor (2008b) and Tsoukis and Miller (2003). They are also directly implied by Barro (1990) and Futagami et al. (1993) who first presented endogenous growth models with productive public services and public capital, respectively.

One implication of these results is that the optimal level of taxation, the optimal public spending composition, and the efficiency of public spending are not interrelated. This means for instance that τ^* also represents the optimal level of taxation if $\phi_{1,2} \neq \phi^*_{1,2}$ and $\kappa^*_{1,2} < 1$. By contrast, we demonstrate below that this does not necessarily hold under CES technology.

4 Optimal Fiscal Policy with Complementarity under Growth Maximization

This section introduces CES technology to the derivation of optimal fiscal policy under growth maximization. Given that no closed-form solutions are feasible, we now use numerical examples to analyze whether the growth-maximizing fiscal policy parameters are responsive to changes in various *exogenous* model parameters which play no role under C-D technology (i.e. they do not enter (15), (16), (17) and (18)).

Figure 1 plots the growth-maximizing tax rate, τ^*, the growth-maximizing expenditure share of total government revenue allocated to G_1, ϕ_1^*, and the growth-maximizing sub-sectoral share of resources allocated to G_A, ϕ_A, as a function of υ (which determines the elasticity of substitution). The slopes deviate from zero, and Figure 1 suggests that τ^* and $\phi_{1,2}^*$ are highly sensitive to the choice of the elasticity of substitution. In addition, with $\upsilon < 0$, the stock-flow properties of the public inputs also impact on the growth-maximizing fiscal policy. This can be seen by noting that even though α_2 (the share parameter associated with G_2) exceeds α_1, the optimal expenditure share ϕ_1^* may exceed 0.5 (and hence ϕ_2^*) when $\upsilon < 0$. In contrast, when Cobb-Douglas technology is assumed and when $\alpha_2 > \alpha_1$, (16) implies that $\phi_2^* > \phi_1^*$ always holds. This is one example of potentially misleading implications of assuming Cobb-Douglas technology. The intuition here is that the level of G_2 (a stock variable) is typically higher than the level of G_1 (which is only derived from the flow of public spending). With complementarity, it is then optimal to increase the share of public resources allocated to G_1 and to increase overall public revenue through higher taxation. Both measures serve to increase the level of G_1. In contrast, the optimal sub-sectoral allocation represented by ϕ_A^* does not respond to exogenous changes in υ because G_A and G_B are both associated with the flow of public spending.

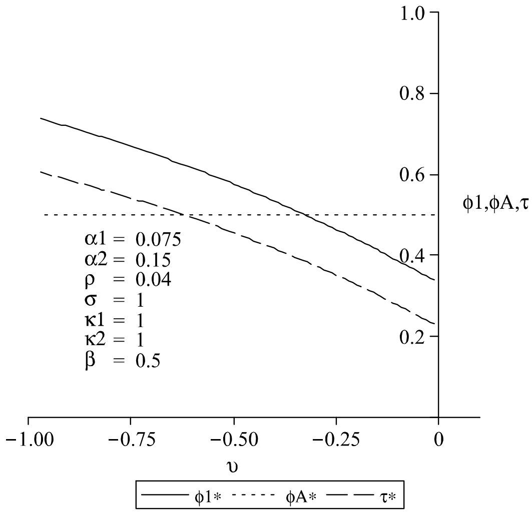
Figures 2 and 3 plot the growth-maximizing tax rate, τ^*, and the growth-maximizing expenditure shares, ϕ_1^* and ϕ_A^*, as a function of σ (which determines the households' inter-temporal elasticity of substitution) and the discount rate parameter ρ. Again, the slopes are non-zero, and it can be seen that with CES technology, both preference parameters determine τ^* and ϕ_1^*. While Figures 2 and 3 suggest that the sensitivity of τ^* and ϕ_1^* to changes in σ and ρ is limited - the slope is not steep - nevertheless this non-zero result is novel.

Under welfare maximization it seems plausible that household preferences would affect optimal fiscal policy. However, under growth maximization, this is less intuitive because, with growth maximization, fiscal policy only directly impacts on private production and income (and not utility). Intuitively, this result follows directly from the model assumptions of complementarity and the fact that the government accumulates public capital.

Complementarity essentially implies that in addition to the share parameters of the production function and the cost of generating public revenue, it is the level of private capital which determines the optimal level of the public inputs. However, the government is unable to manipulate the stock of public capital directly because unlike public services, it is not derived from the flow of public spending but rather accumulated over time similarly to private capital. The growth-maximizing rate of public investment therefore depends on the rate of private investment which, in turn, can be shown to depend on preference parameters. This ensures that the level of public capital depends on the level of private capital as dictated by complementarity. By contrast, the optimal sub-sectoral allocation represented by ϕ_A^* does not respond to exogenous changes in σ and ρ. Thus the allocation of public resources between the two public services depends solely on share parameters in the production function even with CES technology.

These results stress that even within simple models and under the simplifying assumption of growth maximization as the government objective, CES technology significantly changes the nature and determinants of growth-maximizing policy. In particular, the results show that Cobb-Douglas technology understates the complexity of growth-maximizing fiscal policy in the sense that it suggests that growth-maximizing fiscal policy is only determined by particular production technology parameters.

Figure 1: Optimal fiscal policy as a function of υ

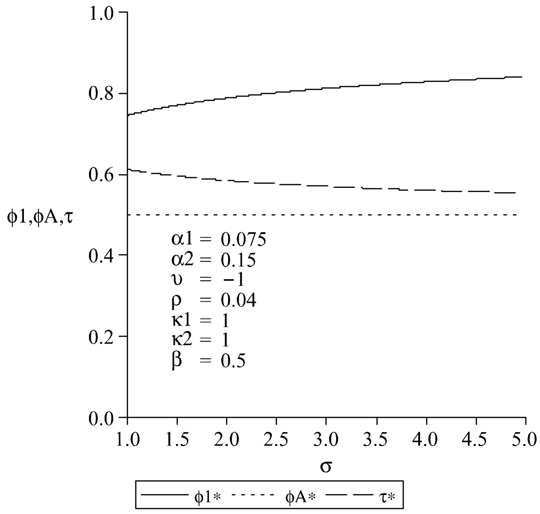


5 Optimal Fiscal Policy with Complementarity under Growth Maximization and Constraints on Government

This section considers the case when governments are constrained in their ability to alter either total public spending or its mix, due, for example, to quasi-fixed expenditure items such as social welfare benefits linked to entitlement conditions, interest payments that depend on the previously accumulated stock of public debt, or wages of public employees. On the revenue side, similar constraints may arise due to the difficulties governments in developing countries experience in taxing large informal sectors, which then limits governments' ability to increase public revenue levels. Given the nature of the underlying legal, economic and political causes of such constraints, it seems plausible to model them as persisting into the long run. In some ways, this assumption is similar to that of García Peñalosa and Turnovsky (2005) who show that 'enforcement problems', as an alternative constraint on the government's budget, alter optimal fiscal policy.[10]

[10]Standard although implicit constraints that are often imposed on governments in endogenous growth models with public finance include the assumptions that lump-sum

Figure 2: Optimal fiscal policy as a function of σ

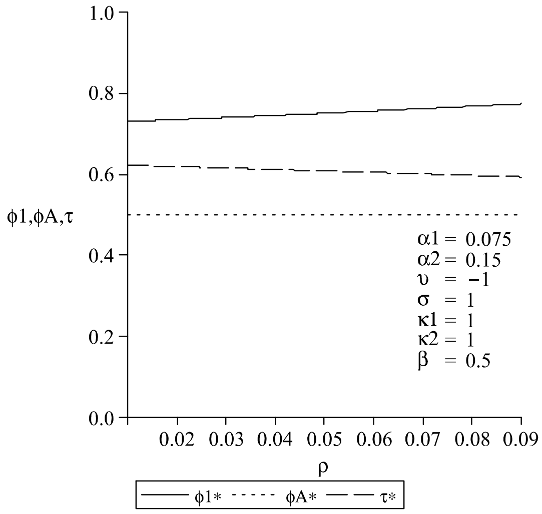


Here, we assume that these constraints limit the ability of governments to set fiscal policy parameters optimally; for simplicity, we model this as the government being unable to adjust one or more fiscal policy parameters, which is then exogenously given. We consider four distinct situations; in each of them one of the four types of fiscal policy parameters in the model is exogenously given and cannot be adjusted by the government due to such constraints. The four parameters we distinguish are: the technical efficiency of public spending determined by κ_i, the rate of taxation τ, the inter-sectoral allocative efficiency determined by ϕ_i, and the sub-sectoral efficiency of public spending determined by ϕ_j.

In the first three cases, we abstract from sub-sectoral allocation within G_1 and set $\beta = 0$ and $\phi_A = 1$ for simplicity. In scenario 1, the technical efficiency of public spending on G_1 is fixed at $\kappa_1 < 1$, whereas τ and $\phi_{1,2}$ are freely adjustable. In scenario 2, the level of taxation is exogenously given and possibly suboptimal so that $\tau \neq \tau^*$ whereas public spending is fully efficient ($\kappa_{1,2} = 1$) in a technical sense given that there is no cost to raise efficiency, and the government sets the expenditure shares $\phi_{1,2}$ optimally. In scenario 3,

taxation is not available and that economic agents take taxes and public spending as given.

Figure 3: Optimal fiscal policy as a function of ρ



the expenditure shares of G_1 and G_2 in total public revenue are exogenously given and possibly suboptimal so that $\phi_{1,2} \neq \phi^*_{1,2}$ whereas public spending is fully efficient in a technical sense ($\kappa_{1,2} = 1$) and τ is freely adjustable. In scenario 4, we set $0 < \beta < 1$ and assume that the sub-sectoral expenditure shares of G_A and G_B in spending on G_1 are exogenously given and possibly suboptimal so that $\phi_{A,B} \neq \phi^*_{A,B}$ whereas public spending is fully efficient ($\kappa_{1,2} = 1$) in a technical sense and τ as well as $\phi_{1,2}$ are freely adjustable.

As discussed above, (15) and (16) imply that under Cobb-Douglas technology, the growth-maximizing tax rate τ^* and the growth-maximizing expenditure shares $\phi^*_{1,2}$ and $\phi^*_{A,B}$ are independent of each other in the sense that deviations from the growth-maximizing tax rate have no impact on the growth-maximizing spending shares and vice versa. In addition, the sub-sectoral public resource allocations and the technical efficiency of public spending neither affect the optimal taxation nor the optimal inter-sectoral public spending composition.

With CES technology, these results fundamentally change even in these simplified cases: the optimal tax rate and expenditure shares in the absence of constraints on government which we refer to as 'first-best', τ^* and $\phi^*_{1,2}$, are not necessarily identical to their optimal values in the presence of constraints referred to as 'second-best' and denoted by τ^{**} and $\phi^{**}_{1,2}$, respectively. As closed-form solutions for the optimal policy parameters are not available with public capital, public services and CES production technology for the market economy, we again resort to numerical examples to show that the value of τ^{**} (ϕ_1^{**}) is responsive to changes in $\kappa_{1,2}$, ϕ_1 and ϕ_A (to changes in $\kappa_{1,2}$, τ and ϕ_A). Figures 4, 5, 6 and 7 represent the four distinct scenarios described above.

Figure 4 captures scenario 1 and plots the second-best values of τ and $\phi_{1,2}$ as a function of the efficiency parameter κ_1 which is exogenously given and which varies between 0.5 and 1. It demonstrates that when $\kappa_1 < 1$, the second-best tax rate, τ^{**}, and the optimal share of resources allocated to G_1, ϕ_1^{**}, exceed the first-best tax rate, τ^*, and the first-best value of ϕ_1, ϕ_1^*, respectively. The intuition is that with complementarity of the inputs to private production, higher levels of taxation and increased resources allocated to G_1 serve to compensate for low public spending efficiency and thereby prevent the levels of G_1 from falling inefficiently low. This is a standard second-best result: replicating first-best policies in a second-best situation may not be optimal. It can also be shown that the growth rate is still lower and does not attain its first-best value.

Figure 5 is based on scenario 2 and plots the second-best value of ϕ_1, ϕ_1^{**}, as a function of τ which is exogenous and varies between 0 and 1 so that it may deviate from τ^*. It likewise demonstrates that when $\tau \neq \tau^*$, the optimal share of public resources allocated to G_1 (the optimal share of public resources allocated to public investment, $\dot{G}_2$) exceeds (falls short of) the one in first-best situation; hence $\phi_1^{**} > \phi_1^*$ ($\phi_2^{**} < \phi_2^*$). The intuition is as follows. G_2 represents the stock of public capital. Current public spending only affects the additions to the stock of capital and but not the existing stock of public capital. When $\tau < \tau^*$ and $\phi_1 = \phi_1^*$, G_1 drops relatively more than G_2.

With complementarity, it is then efficient to allocate a larger share of public resources to G_1 to mitigate the decrease in overall public resources available. In the opposite case, when $\tau > \tau^*$ and $\phi_1 = \phi_1^*$, the intuition is less clear. Given the increase of public resources, the levels of G_1 and of G_2

are higher compared to the first-best situation. However, as G_2 is a stock variable, G_2 is higher than G_1. With complementarity between G_1 and G_2, it is hence efficient to allocate a greater share of public resources to G_1 so that $\phi_1^{**} > \phi_1^*$.

Figure 6 is based on scenario 3 and plots the second-best value of τ, τ^{**}, as a function of ϕ_1 which assumes values between 0 and 1 so that it may deviate from ϕ_1^*. It likewise demonstrates that when $\phi_1 \neq \phi_1^*$, the growth-maximizing level of taxation exceeds that in a first-best situation; hence $\tau^{**} > \tau^*$. The intuition is similar to scenario 1 when κ_1 is set below one. Under misallocation of public resources at the sectoral level, the overall effectiveness of public spending decreases. With complementarity between private and public inputs, it is efficient to compensate for this decrease by increasing the level of taxation (and thereby the level of total public spending).

Figure 7 illustrates scenario 4 and plots the second-best values, τ^{**} and ϕ_1^{**}, as a function of ϕ_A which is exogenously given. It demonstrates that under misallocation of resources at the sub-sectoral level ($\phi_A \neq \phi_A^*$), the growth-maximizing level of taxation and the growth-maximizing share of resources allocated to G_1 exceed those in a first-best situation (hence $\tau^{**} > \tau^*$ and $\phi_1^{**} > \phi_1^*$). The intuition is similar to scenario 1: with sub-sectoral misallocation, the supply level of G_1 falls. With complementarity between private and public inputs, it is efficient to compensate for this decrease by increasing the resources available for spending on G_1 through higher taxation and through reallocation between G_1 and G_2.

These results demonstrate that under CES technology with complementary factor inputs, constraints on government have important implications for optimal fiscal policy under growth maximization that differ from the Cobb-Douglas case. With regard to optimal taxation and public spending composition, second-best fiscal policy parameters may significantly deviate from their first-best values.

In our model, public capital may be thought of as representing public infrastructure which is commonly assumed to play an important role in the process of economic development. However, even if the share parameter of public capital in private production significantly differs from zero (i.e.,

$\alpha_2 > 0$), the optimal share of public resources allocated to public capital ($\phi_2 = 1 - \phi_1$) in situations with constraints may still be relatively small relative to α_2 or even close to zero as shown in Figure 5 depending on the rate of taxation which determines the level of public spending. In addition, the share of public resources allocated to public investment depends on the sub-sectoral allocation of public resources within the production of public services (G_1) as demonstrated in Figure 7.

Our results contrast with those of existing papers which do not examine the impact of constraints on government in combination with complementarity on growth-maximizing fiscal policy. The results of Ghosh and Roy (2004) are closest to ours and imply that in a model with Cobb-Douglas technology, public capital and public services, the optimal tax rate depends on the composition of public spending and vice versa under *welfare maximization.* By contrast, this paper considers *growth maximization*, which is not discussed in detail by Gosh and Roy (2004). They also do not analyze optimal fiscal policy where either the tax rate or the composition of public spending is not set at first-best levels, which makes their results difficult to compare with ours.

Figure 4: Optimal fiscal policy as a function of κ_1

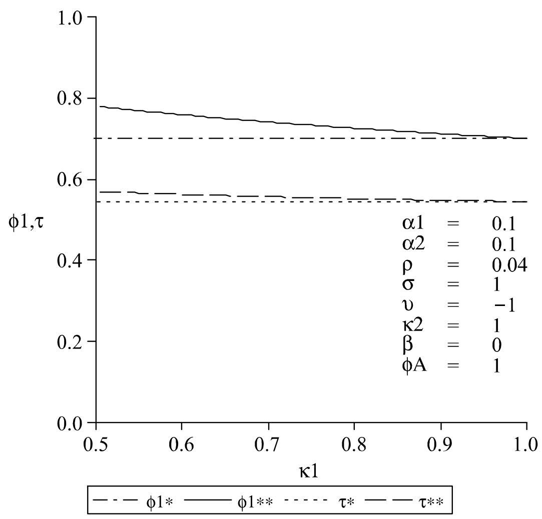
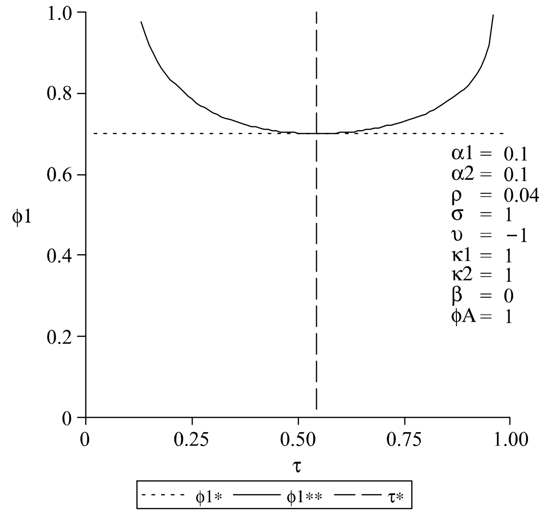


Figure 5: Optimal fiscal policy as a function of τ

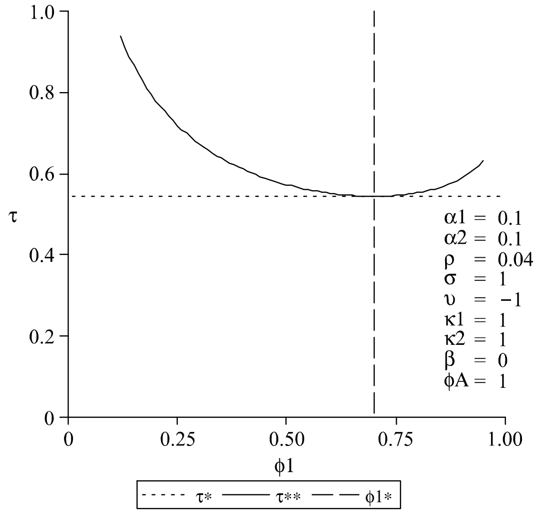


6 Conclusions

This paper has extended standard endogenous growth models with public finance by allowing for input complementarity via CES technology. It has shown that key implications of this class of models with respect to optimal fiscal policy under growth maximization are not robust to these small changes in the underlying assumptions. On the one hand, CES technology implies that the number of model parameters that determine the growth-maximizing fiscal policy significantly increases in otherwise identical models and that preference parameters, the degree of complementarity and the stock-flow properties of public inputs to private production become determinants.

On the other hand, the optimal values of the level of taxation and the composition of public spending are interrelated in the CES case, which is important if the government is constrained in its ability to set one of these parameters optimally. For instance, the optimal share of public investment falls when the level of taxation is not set to its first-best level as a result of constraints. A natural extension would be to derive the welfare-maximizing fiscal policy within the same framework and compare the results to the growth-maximizing equivalent which we leave for future research.

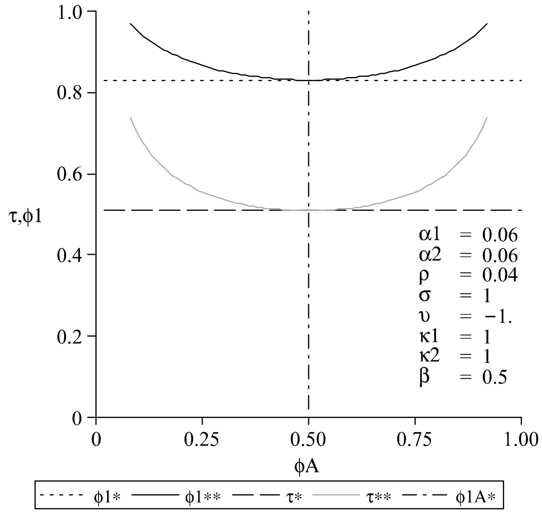
Figure 6: Optimal fiscal policy as a function of ϕ_1



To demonstrate these results, the paper relies on numerical examples. Such an approach is restrictive compared to more general closed-form analytical solutions. Though we have chosen admittedly simple parameter configurations, we would argue that these represent plausible parameter ranges. Further, they serve the purpose of this paper; namely, to show that the model-based conclusions with respect to growth-maximizing fiscal policy that are drawn from models with Cobb-Douglas technology cannot be generalized.

Our primary aim has been to reassess the robustness of the findings of endogenous growth models with respect to the nature of growth-maximizing fiscal policy. Our model is therefore highly stylized but could be extended in future research, for instance by endogenizing labor-leisure choices. While we cannot rule out the possibility that in a more complex model, input complementarity would once again have no effects on growth-maximizing fiscal policy (so that the fiscal policy implications of CES and C-D technology would coincide), our results suggest that such effects are likely to be present with CES technology, and that at least some of such complementarities could be qualitatively and quantitatively important.

Figure 7: Optimal fiscal policy as a function of ϕ_A



A Appendix

A.1 Uniqueness and Stability of the Balanced Growth Path

Let $x = \frac{c}{k}$ and $z = \frac{G_2}{k}$. Together with the transversality condition, $\lim_{t\to\infty}[\lambda k] = 0$, and with the initial conditions, $x_0 > 0$ and $z_0 > 0$, the dynamics of the market economy can be expressed as a system of two differential equations (we assume that $\kappa_i = 1$):

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} \tag{A.1}$$

and

$$\frac{\dot{z}}{z} = \frac{\dot{G}_2}{G_2} - \frac{\dot{k}}{k} \tag{A.2}$$

From (8), (6) and (5), respectively,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left((1-\tau)y_k - \rho\right) \tag{A.3}$$

$$\frac{\dot{k}}{k} = (1-\tau)\frac{y}{k} - x \tag{A.4}$$

$$\frac{\dot{G}_2}{G_2} = \phi_2\tau\frac{y}{G_2} \tag{A.5}$$

Setting $\frac{\dot{x}}{x} = 0$ in (A.1) and solving for x yields its steady state value, $\tilde{x}$:

$$\tilde{x} = (1-\tau)\frac{y}{k} - \frac{1}{\sigma}\left((1-\tau)y_k - \rho\right) \tag{A.6}$$

Using (A.6) to substitute for x in (A.4), and using (A.4) and (A.5) to substitute for $\frac{\dot{k}}{k}$ and $\frac{\dot{G}_2}{G_2}$ in (A.2) yields

$$F = \phi_2 \tau \frac{y}{G_2} - \frac{1}{\sigma}(1-\tau)y_k + \frac{\rho}{\sigma} \tag{A.7}$$

From (4) and (10),

$$\frac{G_1}{G_2} = \frac{\phi_1}{\phi_2}\gamma \tag{A.8}$$

From (1) and (A.8),

$$\frac{y}{G_2} = (\alpha z^{-\upsilon} + \alpha_1 \left(\frac{\phi_1}{\phi_2}\gamma\right)^{\upsilon} + \alpha_2)^{\frac{1}{\upsilon}} \tag{A.9}$$

Differentiating (1) for k, using (4) to substitute for G_1 and replacing $\frac{G_2}{k}$ by z yields

$$y_k = \left(\theta + \alpha_1 \left(\tau\phi_1 \frac{y}{k}\right)^{\upsilon} + \alpha_2 z^{\upsilon}\right)^{\frac{1}{\upsilon}-1} \theta \tag{A.10}$$

From (1) and (4),

$$\frac{y}{k} = \left(\frac{\theta + \alpha_2 z^{\upsilon}}{(1-\alpha_1 \phi_1^{\upsilon} \tau^{\upsilon})}\right)^{\frac{1}{\upsilon}} \tag{A.11}$$

After using (A.11) to substitute in (A.10) and (A.9) and (A.10) to substitute in (A.7), it can be seen that if $\upsilon \leq 0$, $\frac{dF}{dz} < 0$ implying that F is a monotonically decreasing function of z so that there is a unique positive value of $\tilde{z}$ that satisfies $F = 0$. From (A.6), there is a unique positive value of $\tilde{x}$ as well. Thus, the growth path is unique.

To investigate the dynamics in the vicinity of the unique steady state equilibrium, equations (A.1) and (A.2) can be linearized to yield

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x - \tilde{x} \\ z - \tilde{z} \end{bmatrix} \tag{A.12}$$

where $\tilde{x}$ and $\tilde{z}$ denote the steady state values of x and z. From (A.1) and (A.2), $\dot{x}$ and $\dot{z}$ can be rewritten as follows:

$$\dot{x} = \left(\frac{\dot{c}}{c} - \frac{\dot{k}}{k}\right)\tilde{x} \tag{A.13}$$

and

$$\dot{z} = \left(\frac{\dot{G}_2}{G_2} - \frac{\dot{k}}{k} \right) \tilde{z} \tag{A.14}$$

with $\frac{\dot{c}}{c}$, $\frac{\dot{k}}{k}$ and $\frac{\dot{G}_2}{G_2}$ defined according to (A.3), (A.4) and (A.5). Saddlepoint stability requires that the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A.12) must be negative:

$$\det J = a_{11}a_{22} - a_{12}a_{21} \tag{A.15}$$

Given the complexity of the matrix, it is easier to verify numerically that this condition holds. For most sensible examples with sensible parameter values that we used, this condition is satisfied.

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